Q1. If sinθ = $\frac{a^2 - b^2}{a^2 + b^2}$, find cosθ, tanθ and cosecθ.

Q2. ABC is a right angled triangle, in which right angle at angle C, if tan A = 1 then

prove: 2 sin A cos A = 1.

Q3. In a right angled triangle ABC, in which angle B is right angle. If the ratio of sides AB and AC are 1:√2.

Q4. Find the value of $\frac{2\tan A}{1+tan^2 A}$.

Q5. If 16 cot A =12, find the value of $\frac{\sin A + \cos A}{\sin A - \cos B}$

Q6. If tanθ = $\frac{24}{7}$, find the value of sinθ + cosθ.

Q7. If 8 tan A = 15, find the value of sin A – cos B.

Q8. Find the value.

(i) $\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

(ii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

(iii) $\tan^2 10^\circ - \cot^2 80^\circ$

(iv) $\tan^2 30^\circ.\sin 30^\circ + \cos 60^\circ.\sin^2 90^\circ.\tan^2 60^\circ - 2\tan 45^\circ.\cos^2 0^\circ.\sin 90^\circ$

(v) $\frac{\cos 38^\circ \ \mathrm{cosec}\, 52^\circ}{\cot 18^\circ \cot 35^\circ \cot 60^\circ \cot 55^\circ \cot 72^\circ} + \frac{\sin^2 12^\circ + \sin^2 78^\circ}{\sec^2 45^\circ \ \mathrm{cosec}^2 45^\circ}$

(vi) $\sec^2 37^\circ + \cot^2 53^\circ.\tan 21^\circ.\tan 69^\circ - \sin 51^\circ.\cos 49^\circ - \cos 51^\circ.\sin 49^\circ$

Q9. If $\frac{\cos B}{\sin A}$ = n and $\frac{\cos B}{\cos A}$ = m, then show that $(m^2 + n^2)\cos^2 A = n^2$

Q10. Simplify;

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$$

Q11. If tan θ = $\frac{1}{\sqrt{3}}$, find the value of sin (90° - θ).

Q12. Simplify : (1 - sin A) (tan A + sec A)

Q13. If sec θ - tan θ = x, then show that sec θ + tan θ = $\frac{1}{x}$. Find the value of cos θ and sin θ.

Q14. Prove the identities :

$$(1 + \tan^2 A) + \left(1 + \frac{1}{tan^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$$

Q15. Prove the identities :

$$\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}=\left(\frac{1+\sin\theta}{\cos\theta}\right)^2$$

Q16. Prove the identities :

$$\left(\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}\right)\left(\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}\right) = 4 \sec A. \operatorname{cosec} A$$

Q17. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then show that $(m^2 - n^2)^2 = 16\ mn$

Solution: Given that : $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

LHS =

$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta.\sin\theta - [\tan^2\theta + \sin^2\theta - 2\tan\theta.\sin\theta]$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta.\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta.\sin\theta$

$= 2\tan\theta.\sin\theta + 2\tan\theta\ .\ \sin\theta$

$(m^2 - n^2)^2 = [4\tan\theta\ .\ \sin\theta\ ]^2$ .... (1)

RHS = 16 mn

$= 16(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= 16(\tan^2\theta - \sin^2\theta)$

$= 16\ [\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta]$

$= 16\ [\frac{\sin^2\theta - \sin^2\theta\ .\ \cos^2\theta}{\cos^2\theta}]$

$= 16\ [\frac{\sin^2\theta\ (1-\cos^2\theta)}{\cos^2\theta}]$

$= 16\ [\frac{\sin^2\theta\ (1-\cos^2\theta)}{\cos^2\theta}]$

$= 16\ [\frac{\sin^2\theta\ \sin^2\theta)}{\cos^2\theta}]$

$= \left[4\left(\frac{\sin^2\theta)}{\cos\theta}\right)\right]^2$

$= \left[4\frac{\sin\theta}{\cos\theta}.\sin\theta\right]^2$

$16\ mn = [4\tan\theta\ .\sin\theta]^2$ .... (2)

From equation (1) and (2)

$(m^2 - n^2)^2 = 16\ mn$

Q18. If sec θ + tan θ = x, then prove that sin θ = $\frac{x^2-1}{x^2+1}$

Q19. Evaluate :

$$\left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2 + 2\tan 15^\circ \tan 37^\circ \tan 53^\circ \tan 60^\circ \tan 75^\circ$$

Q20. Evaluate :

$$\frac{\cot 5^\circ \cot 10^\circ \cot 15^\circ \cot 60^\circ \cot 75^\circ \cot 80^\circ \cot 85}{(\cos^2 20^\circ + \cos^2 70^\circ) + 2}$$

Q21. Evaluate :

$$\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73} + \sin 25^0 . \cos 65^0 + \cos 25^0 . \sin 65^0 + \sin 30^0$$

Q22. Evaluate :

$$(\operatorname{cosec} 59^\circ . \sec 31^\circ) - \sin 49^\circ \cos 51^\circ + \sin 51^\circ \cos 49^\circ$$

Q23. Evaluate:

$$\frac{\text{Sec} 63^\circ}{\text{Cosec } 27^\circ} + \tan 32^\circ \tan 58^\circ \tan 30^\circ$$

Q24. Evaluate:

$$\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ} + \sin^2 38^\circ \sec^2 52^\circ - \sin^2 45^\circ$$

Q25. Prove the identities :

$$\sqrt{\frac{\operatorname{cosec} \theta + 1}{\operatorname{cosec} \theta - 1}} + \sqrt{\frac{\operatorname{cosec} \theta - 1}{\operatorname{cosec} \theta + 1}} = 2 \sec\theta$$

Q26. Prove that:

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

Q27. Prove that :

$$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

Q28. Prove that :

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

Q29. Prove that:

$$(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

Q30. If $\tan\theta = \frac{a}{b}$, Prove that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Solution: Given that $\tan\theta = \frac{a}{b}$,

$$\text{LHS} = \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$$

Divide numerator and denominator by cos θ

$$\Rightarrow \frac{a\frac{\sin\theta}{\cos\theta} - b\frac{\cos\theta}{\cos\theta}}{a\frac{\sin\theta}{\cos\theta} + b\frac{\cos\theta}{\cos\theta}}$$

$$\Rightarrow \frac{a\tan\theta - b.1}{a\ \tan\theta + b.1}$$

$$\Rightarrow \frac{a.\frac{a}{b} - b}{a.\frac{a}{b} + b}$$

$$\Rightarrow \frac{\frac{a^2 - b^2}{b}}{\frac{a^2 + b^2}{b}}$$

$$\Rightarrow \frac{a^2 - b^2}{a^2 + b^2}$$

LHS = RHS

Q31. If 7 cot θ = 24, prove that $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1}{7}$

Q32. If 4cot θ = 5, show that: $\frac{5\sin\theta + 3\cos\theta}{5\sin\theta - 2\cos\theta} = \frac{1}{7}$

Q33. If $\sec\theta = x + \frac{1}{4x}$, prove that sec θ + tan θ = 2x or $\frac{1}{2x}$

Q34. Evaluate :

$$2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°)$$

Q35. Given that sin (A + B) = sin A cos B + cos A sin B, find the value of sin 75°.

Q36. Given that sin (A - B) = sin A cos B - cos A sin B, find th e value of sin 15°.

Q37. If cos θ + sin θ = $\sqrt{2}$ cos θ, show that cos θ – sin θ = $\sqrt{2}$ sin θ

Solution: cos θ + sin θ = $\sqrt{2}$ cos θ (Given)

$\Rightarrow (\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = 2\cos^2\theta - 1$

Now,

$= \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$

$= 1 - (2\cos^2\theta - 1)$

$= 1 - 2\cos^2\theta + 1$

$= 2 - 2\cos^2\theta$

$= 2(1 - \cos^2\theta)$

$= 2[\sin^2\theta]$ $[\because 1 - \cos^2\theta = \sin^2\theta]$

$(\cos\theta - \sin\theta) = \sqrt{2\sin^2\theta}$

$= \sqrt{2}\sin\theta$

Q38. If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$

Q39. If $a\cos\theta = x$ and $b\cot\theta = y$, show that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

Q40. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = m$ and $\frac{x}{a}\sin\theta + \frac{y}{b}\cos\theta = n$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = m^2 + n^2$

Q41. If $a\cos\theta - b\sin\theta = c$, then prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$

Solution: $(a\cos\theta - b\sin\theta)^2 = c^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta = c^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - c^2 = 2ab\sin\theta.\cos\theta$ .... (i)

Now, $(a\sin\theta + b\cos\theta)^2 = a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta.\cos\theta$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2\sin^2\theta + b^2\cos^2\theta + a^2\cos^2\theta + b^2\sin^2\theta - c^2$ Using (i)

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2\sin^2\theta + b^2\sin^2\theta + a^2\cos^2\theta + b^2\cos^2\theta - c^2$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2\sin^2\theta + a^2\cos^2\theta + b^2\sin^2\theta + b^2\cos^2\theta - c^2$

$\Rightarrow$ (a sin θ + b cos θ)^2 = a^2 (sin^2 θ + cos^2 θ) + b^2 (sin^2 θ + cos^2 θ)- c^2

$\Rightarrow$ (a sin θ + b cos θ)^2 = a^2 (1) + b^2 (1)- c^2 [sin^2 θ + cos^2 θ = 1]

$\Rightarrow$ (a sin θ + b cos θ)^2 = a^2 + b^2 - c^2

$\Rightarrow$ a sin θ + b cos θ = $\pm\sqrt{a^2 + b^2 - c^2}$

Answer:

Q8.

(i) cos θ

(ii)

(iii)

(iv)

(v)

(vi) 0

Q12. cos A

Q13. cos θ = $\frac{2x}{x^2+1}$, sin θ = $\frac{1-x^2}{1+x^2}$